Notre Dame University

Mat 339
Numerical Analysis
Final Exam

Tuesday, January 31st, 2012

Duration: 2 hours

Name: ___________________________________

ID#: ______________________________________

There are 9 problems and 10 pages.
Answer them all.

1)(15%) Newton's method $p_{n+1} = p_n - \dfrac{f(p_n)}{f'(p_n)}$, $n = 0, 1, 2, ...$ for solving the equation $f(x) = 0$ is known to converge quadratically for simple roots of *f*. In this problem, we study the rate of convergence for multiple roots of *f*. Let *p* be a multiple root of $f(x) = 0$ with multiplicity $m \geq 2$.

a) Prove that Newton's method converges linearly.

b) Prove that *p* is a simple root of the function $g(x) = \dfrac{f(x)}{f'(x)}$.

c) Express in terms of f Newton's iteration formula applied to g.

2)(10%) a) Find the polynomial of least degree that assumes these values:

x	-2	-1	0	1	2
y	2	14	4	2	2

b) Deduce the polynomial of least degree that assumes these values:

x	-2	-1	0	1	2	3
y	2	14	4	2	2	10

3)(10 %) The following table for $f(x)$ is given:

x	1.0	1.1	1.2	1.3	1.4
$f(x)$	1.543	1.668	1.810	1.970	2.150

Use all data values to find an approximate value of c for which $f(c)=1.75$

4)(10%) Derive an $O(h^4)$ five-point formula to approximate $f'(x)$ that uses $f(x-h)$, $f(x)$, $f(x+h)$, $f(x+2h)$, and $f(x+3h)$. [***Hint:*** Consider the expression $Af(x-h)+Bf(x+h)+Cf(x+2h)+Df(x+3h)$. Expand in fourth Taylor polynomials, and choose *A*, *B*, *C*, and *D* appropriately.]

5)(10%) Suppose that $N(h)$ is an approximation to *M* for every $h > 0$ and that

$$M = N(h) + K_1 h^2 + K_2 h^4 + K_3 h^6 + \cdots$$

for some constants K_1, K_2, $K_3, \ldots$ Use the values $N(h)$, $N\left(\frac{h}{3}\right)$, and $N\left(\frac{h}{9}\right)$ to produce an $O(h^6)$ approximation to *M*.

6)(15%) How many subintervals are needed to approximate $J = \int_0^1 \frac{\sin x}{x}\,dx$ with error not to exceed $\frac{1}{2} \times 10^{-5}$ using the composite trapezoidal rule?

Hint: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \qquad -\infty < x < \infty$

7)(10%)a)Derive a numerical quadrature formula of the form

$$\int_{-2}^{2} |x| f(x) dx \approx Af(-1) + Bf(0) + Cf(1)$$

that is exact for all polynomials of degree ≤ 2.

b) Is the formula you obtained in (a) exact for polynomials of degree greater than 2? Explain.

8)(10%) Derive the midpoint formula $y_{n+2} = y_n + 2hf_{n+1}$ that is used to approximate the solution of the IVP $y' = f(x, y)$, $y(x_0) = y_0$. Apply the formula to the IVP $(x^2 + 1)\frac{dy}{dx} + xy = 0$, $y(0) = 1$, on $[0, 0.4]$, and compare it with the exact solution $y(x) = \frac{1}{\sqrt{x^2 + 1}}$ for $x = 0.2, 0.3$. Use $h = 0.1$ and $y_1 = y(0.1)$.

9)(10%) The ideal gas law is known to be $p\upsilon^{\gamma} = c$ where γ and c are to be determined. An experiment was performed to determine γ and c. Using the least squares method, determine γ and c that fit the following data:

$\upsilon(cm^3)$	50	60	70	80	90
$p(kg/cm^3)$	63.9	52.0	39.9	22.8	16.7